

mathSUX²

Algebra Cheat Sheet

Order of Operations PEMDAS: Properties of Real Numbers:

P=Parenthesis

E= Exponents

M= Multiplication

D= Division

A= Addition

S= Subtraction

Commutative Property: $a + b = b + a$

Associative Property: $(a + b) + c = a + (b + c)$

Distributive Property: $a(b + c) = ab + ac$

Zero Property:

Identity: $a + 0 = a$

Inverse: $a + -a = 0$

$ab = ba$

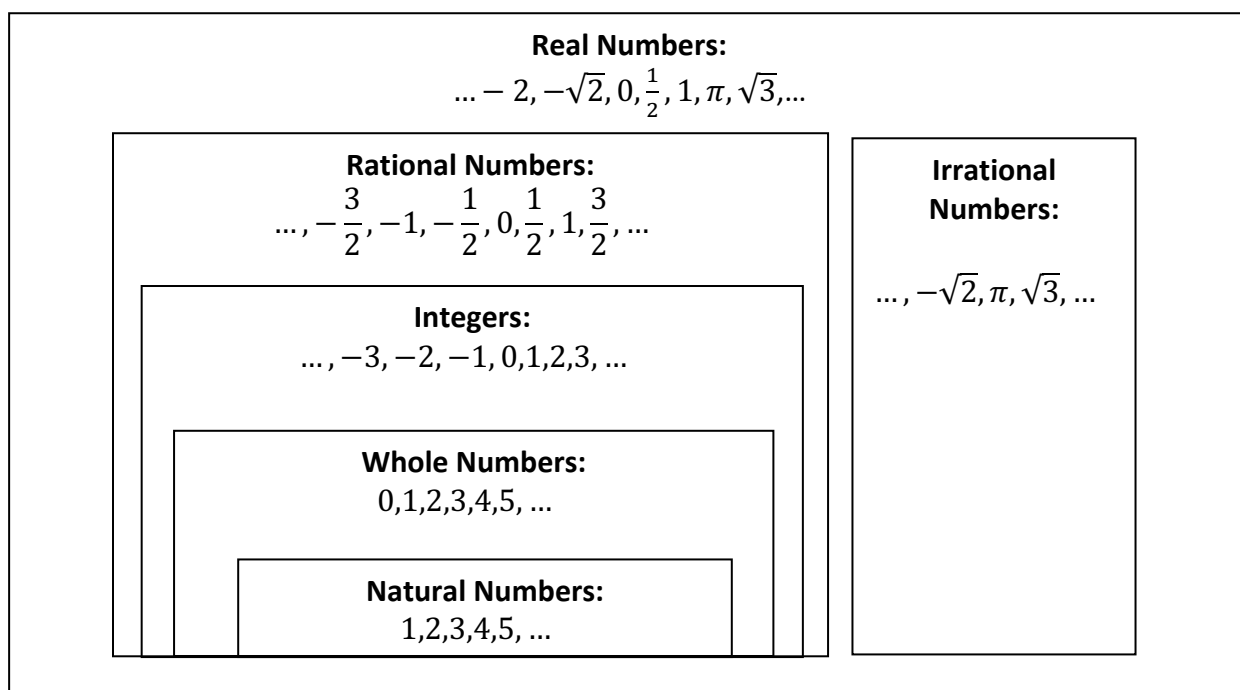
$(ab)c = a(bc)$

$a \times 0 = 0$

$a \times 1 = a$

$a \times \frac{1}{a} = 1$

Real Numbers:



Exponent Rules:

$$x^1 \times x^1 = x^{1+1} = x^2$$

$$\frac{x^1}{x^1} = x^{1-1} = x^0$$

$$(x^3)^2 = x^{3 \times 2} = x^6$$

$$(xy)^2 = x^2y^2$$

Combining Like Terms:

$$x + x = 2x$$

$$3x - x = 2x$$

$$3x^2 + x^2 + 5 = 4x^2 + 5$$

$$5x^2 - 2x^2 + 10x - 5x = 3x^2 + 5x$$

$$10x^2 + 4x - (8x^2 + 2x) =$$

$$10x^2 + 4x - 8x^2 - 2x = 2x^2 + 2x$$

Factoring Methods to Know:

Greatest Common Factor (GCF)

Product/Sum

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Completing the Square

Difference of Two Squares (DOTS)

Functions and their Equations:

Equation of a Line:

$$y = mx + b$$

$m = \text{slope}$

$b = y - \text{intercept}$

Slope Formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{vertical change}}{\text{horizontal change}}$$

Quadratic Equation:

$$ax^2 + bx + c = 0$$

Vertex Formula:

$$x = \frac{-b}{2a}$$

Quadratic Equation in vertex form:

$$y = a(x - h)^2 + k$$

Vertex: (h, k)

Exponential Equation:

$$y = ab^x$$

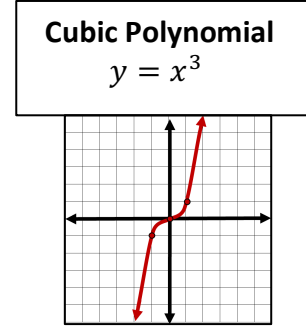
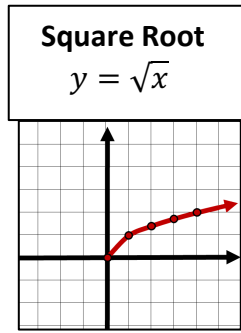
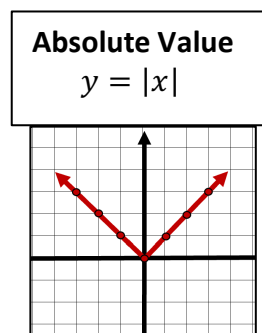
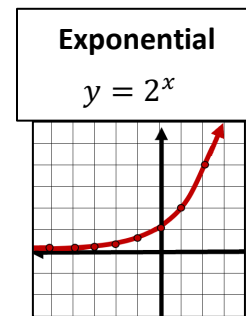
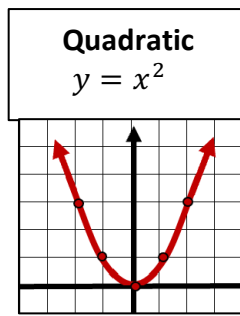
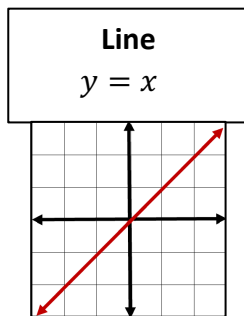
Exponential Growth:

$$b > 1$$

Exponential Decay:

$$0 < b < 1$$

Functions and their Graphs:



Measures of Central Tendency:

Mean: Average

Median: Middle (put numbers in order and find middle)

Mode: "Most" (number that appears the most)

Q1: Quartile 1=Median of first half of data

Q2: Quartile 2=Median of all data

Q3: Quartile 3=Median of second half of data

Box and Whisker Plot:



Geometric Sequence Formula:

(Use when a sequence is formed by multiplying or dividing a number)

$$a_n = a_1 r^{n-1}$$

a_n = Term Value

a_1 = First Term

n = Term Number

r = Common Ratio

Compound Interest Formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

P = Principle

r = Interest rate

n = number of compoundings per year

t = Total number of years

Solve for F: $C = \frac{5}{9}(F - 32)$

$$C = \frac{5}{9}(F - 32)$$

$$\frac{5}{9} \cdot \frac{9}{5} = \frac{5}{9} \cdot \frac{9}{5}$$

$$\frac{5}{5} C = F - 32$$

$$\frac{5}{5} C + 32 = F$$

Arithmetic Sequence Formula:

(Use when a sequence is formed by adding or subtracting a number)

$$a_n = a_1 + (n - 1)d$$

a_n = Term Value

a_1 = First Term

n = Term Number

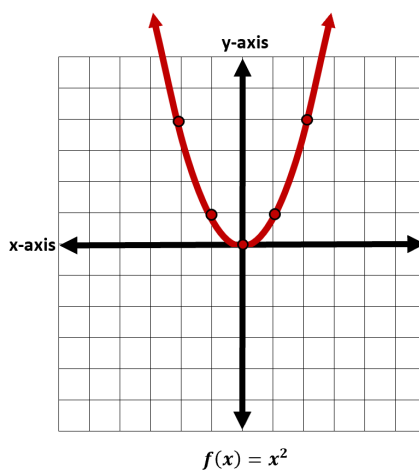
d = Common Difference

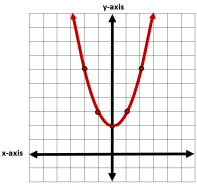
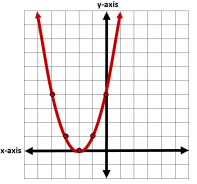
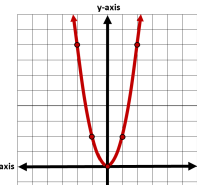
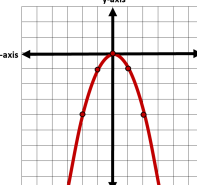
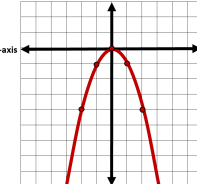
Simplifying Radicals:

$$\sqrt{4} = 2$$

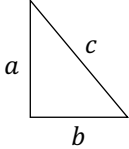
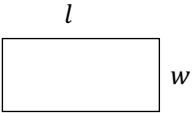
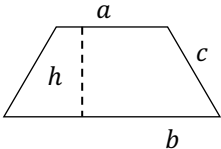
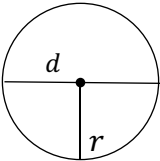
$$\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$$

$$\begin{aligned} 2\sqrt{18} + \sqrt{32} &= 2\sqrt{9 \cdot 2} + \sqrt{16 \cdot 2} \\ &= 2\sqrt{9}\sqrt{2} + \sqrt{16}\sqrt{2} \\ &= 2 \cdot 3\sqrt{2} + 4\sqrt{2} \\ &= 6\sqrt{2} + 4\sqrt{2} \end{aligned}$$

Transformations of a Parabola: $f(x) = x^2$:

Function Transformation	What does it do to the graph?	Graph
$y = f(x) + C$	$C > 0$ moves up $C < 0$ moves down	 $f(x) = x^2 + 2$
$y = f(x + C)$	$C > 0$ moves left $C < 0$ moves right	 $f(x) = (x^2 + 2)$
$y = Cf(x)$	$C > 1$ moves closer to y-axis $0 < C < 1$ moves further from y-axis	 $f(x) = 2x^2$
$y = -f(x)$	Reflection in the x-axis	 $f(x) = -x^2$
$y = f(-x)$	Reflection in the y-axis	 $f(x) = -x^2$

Area and Perimeter:

Shape	Perimeter	Area
Triangle 	$P = a + b + c$	$A = \frac{1}{2}ab$
Square 	$P = 4s$	$A = s^2$
Rectangle 	$P = 2l + 2w$	$A = l \times w$
Trapezoid 	$P = a + b + 2c$	$A = \frac{1}{2}(a + b)h$
Circle 	$C = \pi d$	$A = \pi r^2$

Tips and Tricks!

Circumference: Cherry Pie's Delicious
 $C = \pi d$

Area: Apple Pies Are Two
 $A = \pi r^2$

Basic Properties and Facts

Arithmetic Operations

$$\begin{aligned}
 ab + ac &= a(b + c) & a\left(\frac{b}{c}\right) &= \frac{ab}{c} \\
 \frac{\left(\frac{a}{b}\right)}{c} &= \frac{a}{bc} & \frac{a}{\left(\frac{b}{c}\right)} &= \frac{ac}{b} \\
 \frac{a}{b} + \frac{c}{d} &= \frac{ad + bc}{bd} & \frac{a}{b} - \frac{c}{d} &= \frac{ad - bc}{bd} \\
 \frac{a - b}{c - d} &= \frac{b - a}{d - c} & \frac{a + b}{c} &= \frac{a}{c} + \frac{b}{c} \\
 \frac{ab + ac}{a} &= b + c, \quad a \neq 0 & \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} &= \frac{ad}{bc}
 \end{aligned}$$

Exponent Properties

$$\begin{aligned}
 a^n a^m &= a^{n+m} & (ab)^n &= a^n b^n \\
 (a^n)^m &= a^{nm} & a^0 &= 1, \quad a \neq 0 \\
 \frac{a^n}{a^m} &= a^{n-m} = \frac{1}{a^{m-n}} & \left(\frac{a}{b}\right)^n &= \frac{a^n}{b^n} \\
 a^{\frac{n}{m}} &= \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}} & \frac{1}{a^{-n}} &= a^n \\
 \left(\frac{a}{b}\right)^{-n} &= \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n} & a^{-n} &= \frac{1}{a^n}
 \end{aligned}$$

Properties of Radicals

$$\begin{aligned}
 \sqrt[n]{a} &= a^{\frac{1}{n}} & \sqrt[n]{ab} &= \sqrt[n]{a} \sqrt[n]{b} \\
 \sqrt[m]{\sqrt[n]{a}} &= \sqrt[nm]{a} & \sqrt[n]{\frac{a}{b}} &= \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \\
 \sqrt[n]{a^n} &= a \text{ if } n \text{ is odd} \\
 \sqrt[n]{a^n} &= |a| \text{ if } n \text{ is even}
 \end{aligned}$$

Properties of Inequalities

If $a < b$ then $a + c < b + c$ and $a - c < b - c$

If $a < b$ and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

If $a < b$ and $c < 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

Properties of Absolute Value

$$\begin{aligned}
 |a| &= \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases} \\
 |a| &\geq 0 & |-a| &= |a| \\
 |ab| &= |a| |b| & \left|\frac{a}{b}\right| &= \frac{|a|}{|b|}
 \end{aligned}$$

$$|a + b| \leq |a| + |b| \quad \text{Triangle Inequality}$$

Distance Formula

If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ are two points the distance between them is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Complex Numbers

$$i = \sqrt{-1} \quad i^2 = -1 \quad \sqrt{-a} = i\sqrt{a}, \quad a \geq 0$$

$$(a + bi) + (c + di) = a + c + (b + d)i$$

$$(a + bi) - (c + di) = a - c + (b - d)i$$

$$(a + bi)(c + di) = ac - bd + (ad + bc)i$$

$$(a + bi)(a - bi) = a^2 + b^2$$

$$|a + bi| = \sqrt{a^2 + b^2} \quad \text{Complex Modulus}$$

$$\overline{(a + bi)} = a - bi \quad \text{Complex Conjugate}$$

$$\overline{(a + bi)}(a + bi) = |a + bi|^2$$

Logarithms and Log Properties**Definition**

$y = \log_b(x)$ is equivalent to $x = b^y$

Example

$\log_5(125) = 3$ because $5^3 = 125$

Special Logarithms

$\ln(x) = \log_e(x)$ natural log

$\log(x) = \log_{10}(x)$ common log

where $e = 2.718281828 \dots$

Logarithm Properties

$$\log_b(b) = 1 \qquad \log_b(1) = 0$$

$$\log_b(b^x) = x \qquad b^{\log_b(x)} = x$$

$$\log_b(x^r) = r \log_b(x)$$

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

The domain of $\log_b(x)$ is $x > 0$

Factoring and Solving**Factoring Formulas**

$$x^2 - a^2 = (x + a)(x - a)$$

$$x^2 + 2ax + a^2 = (x + a)^2$$

$$x^2 - 2ax + a^2 = (x - a)^2$$

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

$$x^3 + 3ax^2 + 3a^2x + a^3 = (x + a)^3$$

$$x^3 - 3ax^2 + 3a^2x - a^3 = (x - a)^3$$

$$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$

$$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$$

$$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$$

If n is odd then,

$$x^n - a^n = (x - a)(x^{n-1} + ax^{n-2} + \dots + a^{n-1})$$

$$x^n + a^n = (x + a)(x^{n-1} - ax^{n-2} + a^2x^{n-3} - \dots + a^{n-1})$$

Quadratic Formula

Solve $ax^2 + bx + c = 0$, $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$ – Two real unequal solns.

If $b^2 - 4ac = 0$ – Repeated real solution.

If $b^2 - 4ac < 0$ – Two complex solutions.

Square Root Property

If $x^2 = p$ then $x = \pm\sqrt{p}$

Absolute Value Equations/Inequalities

If b is a positive number

$$|p| = b \Rightarrow p = -b \text{ or } p = b$$

$$|p| < b \Rightarrow -b < p < b$$

$$|p| > b \Rightarrow p < -b \text{ or } p > b$$

Completing the Square

Solve $2x^2 - 6x - 10 = 0$

(1) Divide by the coefficient of the x^2

$$x^2 - 3x - 5 = 0$$

(2) Move the constant to the other side.

$$x^2 - 3x = 5$$

(3) Take half the coefficient of x , square it and add it to both sides

$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2 = 5 + \frac{9}{4} = \frac{29}{4}$$

(4) Factor the left side

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

(5) Use Square Root Property

$$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{2}$$

(6) Solve for x

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

Functions and Graphs**Constant Function**

$$y = a \quad \text{or} \quad f(x) = a$$

Graph is a horizontal line passing through the point $(0, a)$.

Line/Linear Function

$$y = mx + b \quad \text{or} \quad f(x) = mx + b$$

Graph is a line with point $(0, b)$ and slope m .

Slope

Slope of the line containing the two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Slope – intercept form

The equation of the line with slope m and y -intercept $(0, b)$ is

$$y = mx + b$$

Point – Slope form

The equation of the line with slope m and passing through the point (x_1, y_1) is

$$y = y_1 + m(x - x_1)$$

Parabola/Quadratic Function

$$y = a(x - h)^2 + k \quad f(x) = a(x - h)^2 + k$$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at (h, k) .

Parabola/Quadratic Function

$$y = ax^2 + bx + c \quad f(x) = ax^2 + bx + c$$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

Parabola/Quadratic Function

$$x = ay^2 + by + c \quad g(y) = ay^2 + by + c$$

The graph is a parabola that opens right if $a > 0$ or left if $a < 0$ and has a vertex at

$$\left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right).$$

Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Graph is a circle with radius r and center (h, k) .

Ellipse

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Graph is an ellipse with center (h, k) with vertices a units right/left from the center and vertices b units up/down from the center.

Hyperbola

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

Graph is a hyperbola that opens left and right, has a center at (h, k) , vertices a units left/right of center and asymptotes that pass through center with slope $\pm \frac{b}{a}$.

Hyperbola

$$\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$$

Graph is a hyperbola that opens up and down, has a center at (h, k) , vertices b units up/down from the center and asymptotes that pass through center with slope $\pm \frac{b}{a}$.

Common Algebraic Errors

Error	Reason/Correct/Justification/Example
$\frac{2}{0} \neq 0$ and $\frac{2}{0} \neq 2$	Division by zero is undefined!
$-3^2 \neq 9$	$-3^2 = -9$, $(-3)^2 = 9$ Watch parenthesis!
$(x^2)^3 \neq x^5$	$(x^2)^3 = x^2 x^2 x^2 = x^6$
$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$	$\frac{1}{2} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$
$\frac{1}{x^2+x^3} \neq x^{-2} + x^{-3}$	A more complex version of the previous error.
$\frac{a+bx}{a} \neq 1+bx$	$\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$ Beware of incorrect canceling!
$-a(x-1) \neq -ax-a$	$-a(x-1) = -ax+a$ Make sure you distribute the "-".
$(x+a)^2 \neq x^2+a^2$	$(x+a)^2 = (x+a)(x+a) = x^2+2ax+a^2$
$\sqrt{x^2+a^2} \neq x+a$	$5 = \sqrt{25} = \sqrt{3^2+4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3+4=7$
$\sqrt{x+a} \neq \sqrt{x} + \sqrt{a}$	See previous error.
$(x+a)^n \neq x^n+a^n$ and $\sqrt[n]{x+a} \neq \sqrt[n]{x} + \sqrt[n]{a}$	More general versions of previous three errors.
$2(x+1)^2 \neq (2x+2)^2$	$2(x+1)^2 = 2(x^2+2x+1) = 2x^2+4x+2$ $(2x+2)^2 = 4x^2+8x+4$ Square first then distribute!
$(2x+2)^2 \neq 2(x+1)^2$	See the previous example. You can not factor out a constant if there is a power on the parenthesis!
$\sqrt{-x^2+a^2} \neq -\sqrt{x^2+a^2}$	$\sqrt{-x^2+a^2} = (-x^2+a^2)^{\frac{1}{2}}$ Now see the previous error.
$\frac{a}{\left(\frac{b}{c}\right)} \neq \frac{ab}{c}$	$\frac{a}{\left(\frac{b}{c}\right)} = \frac{\left(\frac{a}{1}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{1}\right)\left(\frac{c}{b}\right) = \frac{ac}{b}$
$\frac{\left(\frac{a}{b}\right)}{c} \neq \frac{ac}{b}$	$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$